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# Local geometry of the Fermi surface and quantum oscillations in the linear response of metals

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**Abstract**

In this paper we present a theoretical analysis of the effect of anomalies of the Fermi surface (FS) curvature on oscillations in the electron density of states (DOS) in strong magnetic fields. It is shown that the oscillations could be significantly weakened when the FS curvature takes on zero or extremely large values or diverges in the vicinities of some extremal cross-sections. This leads to a characteristic dependence of the oscillation magnitude on the direction of the magnetic field which is studied. The results of these general studies are employed to analyse the effects of the FS curvature anomalies on the quantum oscillations in the velocity of sound waves travelling in metals.

## 1. Introduction

Experimental data concerning quantum oscillations in various observables in metals under strong magnetic fields have been repeatedly used in studies of their electron characteristics [1]. It is known that the electron response of a metal to an external disturbance depends on the geometry of the Fermi surface (FS). The Fermi surfaces of metals are mostly complex in shape and this can significantly influence observables. At present those phenomena which are determined by the main geometric characteristics of the FSs, i.e. their connectivity, are well studied. However, those effects which appear due to some fine geometrical features of the FSs such as nearly cylindrical segments or points of flattening have not been investigated in detail so far. Meanwhile these elements in the FS geometry can noticeably affect the electron response of metal.

When the FS includes points and/or lines where its curvature becomes zero, this leads to an enhancement of the contribution from the neighbourhood of these points/lines to the electron density of states (DOS) on the FS. Usually this enhanced contribution is small compared to the main term in the DOS which originates from the remaining major part of the FS. Therefore it cannot produce noticeable changes in the response of the metal when all segments of the FS contribute to the response functions essentially equally. However, some effects (including those originating from quantum oscillations in the electron DOS) are determined with small 'effective' segments of the FS. The contribution to the DOS from the vicinities of the points and lines of zero curvature can be congruent to the contribution of an 'effective' segment. In other words, when the curvature of the surface becomes zero at some part on an 'effective' segment of the FS, it can give a sensible enhancement of efficient electrons and, in consequence, a pronounced change in the response of the metal to the disturbance.

It has been shown that when the FS includes points or lines where at least one of the principal radii of curvature becomes infinitely large, then changes may be observed in the frequency and temperature dependences of sound dispersion and absorption [2–6], and in the frequency dependence of the surface impedance of a conventional metal under the anomalous skin effect [7, 8]. Likewise, the flattening points at the FS may give rise to some anomalies in the magnetoacoustic response of a two-dimensional electron gas, as shown in [9, 10].

In the present work we concentrate on quantum oscillations in observables in metals which originate from the DOS oscillations in strong magnetic fields. The oscillating term in the DOS is formed by the neighbourhoods of the FS cross-sections with extremal areas. Usually, the latter are narrow strips on the FS. Therefore the FS curvature anomalies at these strips may significantly affect their contributions to the DOS quantum oscillations. Consequently, the effect of the FS local geometry can be revealed in oscillations of various observables in metals, such as magnetization, magnetic susceptibility, resistivity, sound velocity and attenuation. Qualitative anomalies of the de Haas–van Alphen oscillations associated with cylindrical pieces of the FSs were considered before in [11–13]. Also, it was shown that nearly cylindrical pieces inserted in the FS could cause noticeable softening of some acoustic modes near the peaks of the DOS quantum oscillations (see [14, 15]).

Along with the points and lines of zero curvature there can exist points and lines where the FS curvature diverges. A simple illustration is a kink line on a surface where the electron velocity is discontinuous. If the FS inserts such lines, then the effective belts could be missed for appropriate directions of propagation of electromagnetic or sound waves, which brings anomalies in high-frequency properties of the metal. It is shown [16, 17] that under these conditions electromagnetic waves of some special kind can propagate in metals. The lines of a singular (infinite) curvature can be arranged on edges of narrow lenses or needle-shaped cavities, which are elements of the FSs of some metals. However, it could happen that the FS curvature diverges at some points where the velocity of electrons varies continuously, as discussed below.

A characteristic feature of quantum oscillations attributed to the extremal cross-sections of anomalous curvature is a clearly manifested dependence of their amplitude on the direction of an external magnetic field **B**. The effect could be revealed only for certain directions, when the effective cross-section runs along that part of the FS where the curvature becomes zero or diverges. When the magnetic field is tilted away from such a direction by an angle $\Phi$ the effective cross-section slips from the 'anomalous' segment of the FS. This brings measurable changes in the oscillation magnitude. This is typical for all effects arising due to local peculiarities in the FS geometry. For instance, the angular dependence of the amplitudes of magnetoacoustic commensurability oscillations in a two-dimensional electron system was discussed in [10]. In the present work we analyse the angular dependences of quantum oscillation magnitudes for different kinds of local anomalies of the FS curvature. The results could be employed to analyse manifestations of the FS local geometry in quantum oscillations of various characteristics in metals. As an example, we briefly consider the effect of the FS curvature anomalies on quantum oscillations in the velocity of an ultrasonic wave propagating in a metal. To avoid further prolixities in this work, other applications are not included. These applications will be discussed elsewhere.

## 2. The model

The concept of a zero curvature line (nearly cylindrical cross-section) on the FS does not require further explanation. In contrast, the concept of the line where the FS curvature diverges needs clarification. Before introducing the FS model used in the following analysis, we present a simple illustration to clear this concept.

It is known that FSs of cadmium and zinc (both metals have an hcp crystalline structure) include an axially symmetric electron lens. The lens is located at the centre of third Brillouin zone, with its axis running along the [0001] direction. We can roughly reproduce this lens using the Harrison method of construction of free electron FSs. Assuming that the $z$ axis of the coordinate system is directed along the axis of symmetry, we arrive at the following energy–momentum relation for electrons associated with the lens:

$$E(\mathbf{p}) = \frac{\mathbf{p}_\perp^2}{2m} + \frac{p_m^2}{2m}\left(\mathrm{sgn}(x)\cdot x + \frac{hG}{2p_m}\right)^2. \tag{1}$$

Here, $\mathbf{p}_\perp$ and $p_z$ are the electron quasimomentum components across and along the symmetry axis, respectively; $m$ is the electron effective mass; $x = p_z/p_m$, where $p_m$ is the maximum value of the quasimomentum along the $z$ axis; and $\mathbf{G} = (0, 0, G)$ is the corresponding reciprocal lattice vector.

The lens described with (1) has a kink line running along its edge. However, this curvature anomaly appears owing to our free electron approximation. In actual metals we can expect it to be smoothed by means of crystalline fields. To better reproduce the shape of the lens near the edge we start from the standard asymptotic for the sign function which is valid for $|x| < 1$, namely

$$\mathrm{sgn}(x) = \lim_{l\to\infty} f(x,l) \equiv \lim_{l\to\infty} |x|^{1/l+1}/x. \tag{2}$$

To proceed we use the approximation $\mathrm{sgn}(x) \approx f(x,l)$. Substituting this approximation into (1) we see that now the velocity of the electrons belonging to the lens varies continuously, and its longitudinal component reduces to zero at the central cross-section ($p_z = 0$). Keeping only the greatest terms, we can write the following expression for the cross-sectional area near the central cross-section:

$$A(x) = A_{\mathrm{ex}}(1 - b^2|x|^{2k}). \tag{3}$$

Here, $A_{\mathrm{ex}}$ is the central cross-sectional area, $b^2$ is a dimensionless positive constant, and $k = \frac{1}{2}(1 + 1/l)$. The lens curvature near the central cross-section could be approximated as follows:

$$K(x) \approx -\frac{1}{2p_m^2 A_{\mathrm{ex}}}\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} \sim \frac{|x|^{2k-2}}{p_m^2}. \tag{4}$$

Since $k < 1$, the curvature tends to infinity when $p_z$ tends to zero although there is no break in the longitudinal velocity at $p_z = 0$, as shown in the figure 1. This means that the central cross-section of our FS is not a kink line, and the effective strip still exists here. However, there could be a considerable decrease in the number of electrons associated with the vicinity of this cross-section. It can influence some properties of the metal, including quantum oscillations. Actually, there exists a piece of experimental evidence for the curvature anomaly at the edge of the electron lens in cadmium. A resonance feature at the cyclotron frequency was observed in the surface impedance derivative when the magnetic field was directed along the [0001] axis [18]. This resonance originates from the contribution of the electrons associated with the edge of the lens, and it could be explained assuming that the lens curvature diverges there [19].

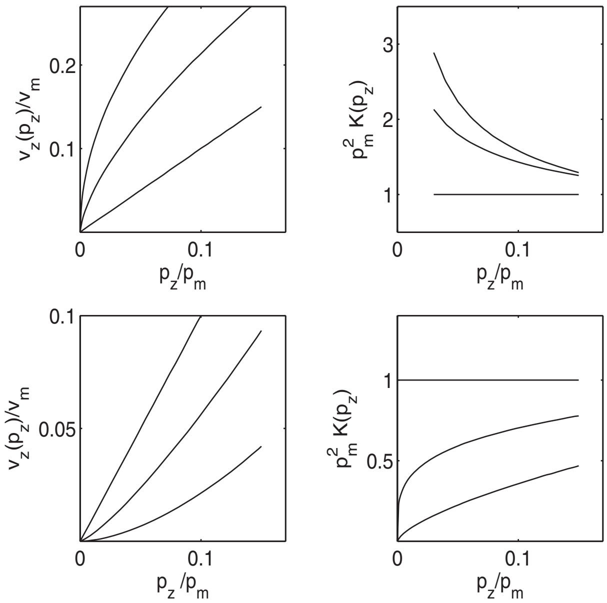


**Figure 1.** Curvature anomalies at an axially symmetric Fermi surface. Top panels: plots of $v_z$ versus $p_z$ (left panel) and $K$ versus $p_z$ (right panel) in the vicinity of the extremal cross-section at $p_z = 0$ where the FS curvature $K$ diverges. The curves are plotted according to equations (4) and (5) for $m_\perp = m_\|$; $k = 3/4, 5/6, 1$ from the top to the bottom. Bottom panels: plots of $v_z$ versus $p_z$ (left panel) and $K$ versus $p_z$ (right panel) in the vicinity of the nearly cylindrical cross-section at $p_z = 0$. The curves are plotted for $m_\perp = m_\|$; $k = 1, 9/8, 4/3$ from the top to the bottom.

In further calculations we use a simple model of a closed simply connected and axially symmetric FS which has a unique extremal cross-section for any direction of the external magnetic field. Such an FS could be described with the energy–momentum relation

$$E(\mathbf{p}) = \frac{\mathbf{p}_\perp^2}{2m_\perp} + \frac{p_m^2}{2m_\|}|x|^{2k}. \tag{5}$$

When $k = 1$, equation (5) corresponds to the ellipsoidal FS. In this case $m_\perp, m_\|$ are the principal values of the effective mass tensor. As previously, the cross-sectional area of the FS near its central cross-section is described with the expression (3), where $b^2 = 1$. It follows from (4) that at $\Phi = 0$ (the magnetic field is directed along the symmetry axis), the chosen FS is nearly cylindrical in the vicinity of the central cross-section when $k > 1$. However, for $1/2 < k < 1$ the FS curvature diverges here. Suppose that the magnetic field $\mathbf{B}$ deviates from the symmetry axis by the angle $\Phi$. Then the expression for the cross-sectional area changes. After some straightforward calculations we arrive at the result:

$$A(x) = A_{\text{ex}}(1 - a^2x^2 - b^2x^{2k}); \qquad k > 1/2. \tag{6}$$

Here, $a^2, b^2$ and $A_{\text{ex}}$ are functions of the angle $\Phi$ between the magnetic field and the FS symmetry axis, $a^2(0) = 0$, and it increases as $\Phi$ increases. When the angle takes on a certain value $\Phi_0$, $a^2(\Phi)$ becomes greater than $b^2(\Phi)$, and stays likewise when $\Phi$ further increases.

## 3. The effect of the Fermi surface curvature anomalies on the quantum oscillations in the electronic density of states

For FSs with a single extremal cross-section the electron DOS is given by

$$N_\zeta \equiv -\sum_\nu \frac{\mathrm{d} f_\nu}{\partial E_\nu} = g(1+\Delta). \tag{7}$$

Here, $f_\nu$ is the Fermi distribution function for quasiparticles with energies $E_\nu$; $g$ is the value of the electron DOS in the absence of the magnetic field, and the oscillating function $\Delta$ is described by the expression

$$\Delta = \sum_{r=1}^{\infty} (-1)^r \psi_r(\theta) \cos\left[\pi r \frac{m_\perp}{m}\right] \int_0^1 \cos\left[\frac{rcA(x)}{\hbar|e|B}\right] \mathrm{d}x, \tag{8}$$

where $\psi(\theta) = r\theta/\sinh r\theta$; $\theta = 2\pi^2 T/\hbar\Omega$; $\hbar\Omega$ is the cyclotron quantum; $T$ is the temperature expressed in energy units.

Substituting (6) into (8), we obtain

$$\Delta = \sum_{r=1}^{\infty} (-1)^r \psi_r(\theta) \cos\left(\pi r \frac{m_\perp}{m}\right) \left[\cos(\pi r \gamma^2) U_r(\Phi) + \sin(\pi r \gamma^2) W_r(\Phi)\right]. \tag{9}$$

Here,

$$U_r(\Phi) = \int_0^1 \cos\left[\pi r \gamma^2 (a^2 x^2 + b^2 x^{2k})\right] \mathrm{d}x, \tag{10}$$

$$W_r(\Phi) = \int_0^1 \sin\left[\pi r \gamma^2 (a^2 x^2 + b^2 x^{2k})\right] \mathrm{d}x. \tag{11}$$

In further analysis we assume that the number of Landau levels under the FS is large, and the parameter $\gamma^2 = 2\zeta/\hbar\Omega$ ($\zeta$ is the chemical potential of electrons) takes values much larger than unity. So, we can derive asymptotic expressions for the integrals (10), (11) using the stationary phase method.

The principal terms in the expansions of these integrals in powers of the small parameter $\gamma^{-1}$ are easily obtained in two limiting cases: $a^2 \ll b^2$ and $a^2 \gg b^2$. The first inequality is satisfied for small values of the angle between the magnetic field and symmetry axis, when the extremal cross-section of the FS nearly coincides with the line of anomalous curvature. The asymptotic expression for the oscillating function $\Delta$ has the form

$$\Delta = \frac{\eta_k}{\gamma^{1/k}} \sum_{r=1}^{\infty} \frac{(-1)^r}{r^{1/2k}} \psi_r(\theta) \cos\left[\frac{rcA_{\mathrm{ex}}}{\hbar|e|B} - \frac{\pi}{4k}\right] \cos\left[\pi r \frac{m_\perp}{m}\right], \tag{12}$$

where

$$\eta_k = \frac{\Gamma(1/2k)}{2k(b\sqrt{\pi})^{1/k}} \frac{m_\perp p_m}{\pi^2 \hbar^3 g}.$$

In the opposite limit $a^2 \gg b^2$, which corresponds to sufficiently large values of the angle $\Phi$, the principal term in the expansion of $\Delta$ in powers of $\gamma^{-1}$ equals

$$\Delta = \frac{1}{\gamma} \sum_{r=1}^{\infty} \frac{(-1)^r}{\sqrt{r}} \psi_r(\theta) \cos\left[\frac{rcA_{\mathrm{ex}}}{\hbar|e|B} - \frac{\pi}{4}\right] \cos\left[\pi r \frac{m_\perp}{m}\right]. \tag{13}$$

So, in this case, the function $\Delta$ describes ordinary oscillations of the electron density of states in a quantizing magnetic field.

It is well known that the amplitudes of quantum oscillations depend on temperature. Ordinary quantum oscillations of the electron DOS described by the equation (13) have

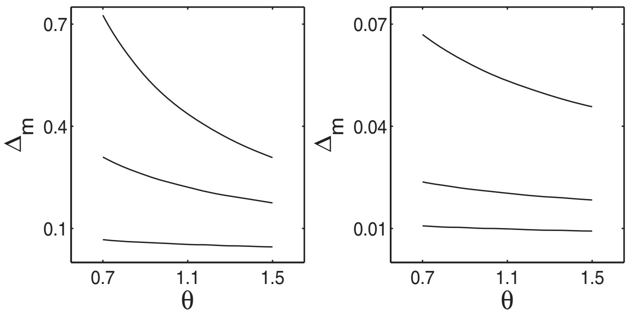


**Figure 2.** Effect of the FS curvature on the temperature dependence of quantum oscillation amplitudes at moderately low temperatures ($\theta \sim 1$). Left panel: nearly cylindrical cross-sections; $k = 4, 2, 1$ from the top to the bottom. Right panel: cross-sections where the FS curvature diverges; $k = 1, 3/4, 5/8$ from the top to the bottom. Curves are plotted assuming $m_\parallel = m_\perp$, $\pi\gamma^2 = 10^3$.

amplitude of the order of $\gamma^{-1/2}\theta^{-1/2}$. The amplitude of the oscillations related to the 'anomalous' cross-section given by (12) may be estimated as $\gamma^{-1/k}\theta^{(1-2k)/2k}$. Comparing these estimates, we see that the amplitude of oscillations associated with the extremal cross-section of zero/diverging curvature differs significantly from that of the usual quantum oscillations. The contribution from a nearly cylindrical cross-section can considerably exceed in amplitude contributions from usual cross-sections. In addition, the amplitude of DOS oscillations originating from a cross-section where the FS curvature diverges is smaller than that of the ordinary oscillations. Also, the fine geometrical structure of the FS near the extremal cross-sections may bring changes in the temperature dependence of the oscillation amplitude. As shown in figure 2, the closer the vicinity of an extremal cross-section is to a cylinder, the more pronounced is the decrease in the oscillation amplitude while the temperature rises. Oscillations associated with a cross-section where the FS curvature diverges exhibit weaker temperature dependence compared to ordinary quantum oscillations.

Now we analyse the amplitude of oscillations within the intermediate range where $a^2$ and $b^2$ have the same order of magnitude. We present the integrals (10) and (11) as expansions in powers of the parameter $w$ ($w = \pi r\gamma^2 a^2/(\pi r\gamma^2 b^2)^{1/k}$). In particular, the expression for $U_r$ in the case $w < 1$ has the form

$$U_r(\omega) = \frac{(\pi r\gamma^2 b^2)^{-1/2k}}{2k}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}w^n\Gamma\left(\frac{2n+1}{2k}\right)\cos\left[\pi\frac{n(1-k)}{2k}+\frac{\pi}{4k}\right], \tag{14}$$

while for $w > 1$ we arrive at the following expansion:

$$U_r(\omega) = \frac{1}{2\pi a\gamma\sqrt{r}}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}w^{-nk}\Gamma\left(kn+\frac{1}{2}\right)\cos\left[\pi\frac{n(k-1)}{2}+\frac{\pi}{4}\right]. \tag{15}$$

Expansions for $W_r$ could be obtained from (14) and (15) by replacing cosines with sines of the same arguments.

The results of computation of the oscillation amplitude $Y$ ($Y = \sqrt{U^2+W^2}$) as a function of $w$ are presented in figure 2. Calculations were carried out assuming that $\theta \approx 1$. Accordingly, only the first term was kept in the sum over $r$ included in (12). The dependence of $Y$ on $w$ calculated for $k = 2$ and 4 is shown in the left panel. In this case the central cross-section of the FS for $\Phi = 0$ is quasi-cylindrical. The oscillation amplitude takes on the largest value when $\Phi = 0$ and decreases monotonically as $w$ increases. Both curves are asymptotically approaching the straight line $Y = 1/\gamma$ when $w \gg 1$. As expected, the amplitude variation is much better pronounced for curve 2. This curve is plotted for the larger value of the parameter $k$

when the FS is closer to a cylinder near its central cross-section. In the right panel the behaviour of the amplitude of quantum oscillations of the DOS upon a variation of the angle $\Phi$ is shown under the assumption that the FS curvature diverges at the central cross-section at $\Phi = 0$. The curves presented here are plotted for $k = 3/4$ and $5/8$. The second value of $k$ corresponds to a stronger anomaly in the FS curvature at the extremal cross-section. For $k = 5/8$, the oscillation amplitude increases monotonically with $w$ and tends to the value $1/\gamma$. For a less pronounced anomaly in the curvature ($k = 3/4$), the dependence of $Y$ on $w$ is nonmonotonic. However, in this case the decrease in the value of $Y$ for small values of $w$ is replaced by its increase when $w$ takes on greater values.

Now, we remark that two parameters characterizing the FS local geometry were used in the present work, and they both could be easily evaluated provided that we possess relevant experimental data. The angle $\Phi_0$ describing the width of the FS strip where the curvature anomalies are revealed is a directly measurable quantity, and the shape parameter $k$ introduced in equations (3) and (5) could be estimated based on the variation in the oscillation magnitude between its maximum/minimum ($\Phi = 0$) and plateau ($\Phi \gg \Phi_0$) values. The magnitudes of oscillations described by equations (12), (13) have the order of $\gamma^{-1/k}\theta^{(1/2k)-1}$, and $\gamma^{-1}\theta^{-1/2}$, respectively. So, the ratio of maximum/minimum and plateau magnitudes $\rho$ takes on the value $\rho \sim \gamma^{1-1/k}\theta^{(1-k)/2k}$, which gives the following estimate for the shape parameter: $k \approx (\frac{1}{2}\ln\theta - \ln\gamma)/(\frac{1}{2}\ln\theta - \ln\gamma + \ln\rho)$.

## 4. An application: quantum oscillations in the ultrasound velocity

To make the presented results more specific we apply our analysis to study the effects of the FS local geometry on the quantum oscillations of the velocity of ultrasound waves travelling in a metal. When an ultrasound wave propagates in a metal the crystalline lattice is periodically deformed. This gives rise to electric fields which influence the electrons. In addition, the periodical deformations of the lattice cause changes in the electronic spectrum. Here, we omit these deformation corrections to simplify further analysis.

The emergence of the electric field accompanying the lattice deformation leads to a redistribution of the electron density $N$. The local change in the electronic density $\delta N(\mathbf{r})$ equals

$$\delta N(\mathbf{r}) = \frac{\partial N}{\partial \zeta} e\varphi(\mathbf{r}) \equiv N_\zeta e\varphi(\mathbf{r}). \tag{16}$$

The relation (17) has to be complemented by the condition of electrical neutrality of the system:

$$\delta N(\mathbf{r}) - eN \operatorname{div} \mathbf{u}(\mathbf{r}) = 0. \tag{17}$$

We use these equations to express the potential $\varphi(\mathbf{r})$ in terms of the lattice displacement vector. As a result we derive the expression for the electron force $\mathbf{F}(\mathbf{r})$ acting upon the lattice under its displacement by the vector $\mathbf{u}(\mathbf{r})$. For a longitudinal ultrasound wave travelling along the magnetic field $\mathbf{B}$, we obtain

$$F(\mathbf{r}) = -\frac{N^2}{N_\zeta}\mathbf{b}_0 \Delta \mathbf{u}(\mathbf{r}). \tag{18}$$

Here, $\mathbf{b}_0$ is a unit vector directed along the field $\mathbf{B}$. More thorough analysis taking into account deformation corrections to the electron energies does not bring qualitative changes in expression (18). We arrive at the modified expression for the force $F(\mathbf{r})$ simply by multiplying (18) by a dimensionless factor of the order of unity which could be treated as a constant.

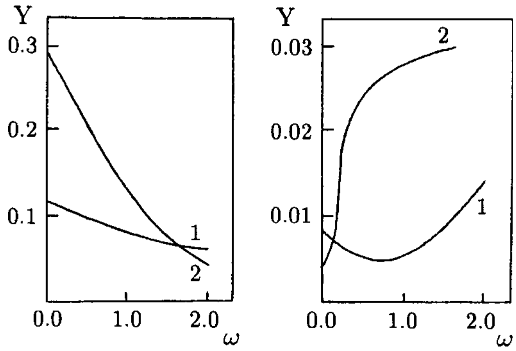


**Figure 3.** The amplitude of the electron DOS quantum oscillations as a function of the magnetic field orientation. Left panel: $k = 2$ (curve 1), $k = 4$ (curve 2). Right panel: $k = 3/4$ (curve 1), $k = 5/8$ (curve 2). The curves are plotted for $\pi\gamma^2 = 10^3$.

Starting from (19) we obtain the well-known result for the oscillating correction $\tilde{s}$ to the sound velocity $s$:

$$\frac{\tilde{s}}{s} = -\frac{N^2}{\rho s^2 g} \sum_i \Delta_i \tag{19}$$

where $\rho$ is the metal density and the summation is carried out over all extremal cross-sections on the FS corresponding to the given direction of the magnetic field. So, the above angular dependences of magnitudes of quantum oscillations arising due to the local geometry of the FS could be revealed in oscillations of the velocity of ultrasound waves propagating in metals.

## 5. Conclusion

In summary, we showed that when the FS of a metal inserts lines where its curvature becomes zero or reveals discontinuity, this can significantly affect both the amplitude and phase of electron DOS oscillations in strong (quantizing) magnetic fields. The effect arises due to an increase/decrease of the relative number of electrons associated with the neighbourhoods of these lines. It reveals itself at certain directions of the magnetic field, and disappears when the magnetic field is tilted away from such a direction. This results in an angular dependence of the DOS oscillation amplitudes. The latter could be observed in experiments on quantum oscillations in various characteristics of a metal such as magnetization, magnetic susceptibility and resistivity, for all these oscillations arise due to the DOS quantum oscillations. Quantum oscillations in the sound velocity discussed in this work is a mere example of the effect of the FS curvature local anomalies on observables.

The most obvious materials where we can expect the FS curvature anomalies to be manifested include layered structures with metallic-type conductivity (e.g. the $\alpha$-(BEDT-TTF)$_2$MHg(SCN)$_4$ group of organic metals). The Fermi surfaces of these materials are sets of rippled cylinders, isolated or connected by links [20]. An angular dependence of the magnetic oscillation magnitude resembling that shown in the figure 3 is well known for such materials. The appearance of this effect shows that the quasi-two-dimensional FSs of some organic metals include segments with zero curvature. The effect was first analysed in [21], and further discussed in some later works (see e.g. [20]).

Local geometry of the FSs of usual metals can also be displayed in the angular dependences of magnitudes of quantum oscillations. For instance, there is an experimental evidence that 'necks' connecting quasispherical pieces of the FS of copper include nearly cylindrical belts [1]. When the magnetic field is directed along the axis of a 'neck' (for instance, along the [111] direction in the quasimomenta space), the extremal cross section of the 'neck' could be expected to run along the nearly cylindrical strip where the FS curvature turns zero. It is also likely that the FS of gold possesses the same geometrical features for it closely resembles that of copper. As for possible divergences of the FS curvature, experiments of [18] give grounds to conjecture that such anomalies could be found on the FSs of cadmium and zinc. It is possible that the curvature is anomalously large at the edge of the electron lens which is the part of the FSs in both metals. For all above listed substances we can expect the effect to be revealed at reasonably low temperatures ($\sim$1 K) and reasonably strong magnetic fields (1–10 T), giving additional information on the fine geometrical characteristics of the FSs.

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